CONSIDERATIONS OF PROBABILITY OF DETECTION IN FRACTURE-CRITICAL INSPECTIONS OF FORGED POLISHED CAR RIMS

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Abstract. The paper begins by discussing some basics of fracture mechanics for the computational modelling. The statistical model termed probability of detection (POD) is used to establish the ability of inspections to detect defects. Probability is the mathematical calculation of the likelihood that an event will occur.

The car rims are made out of strong metals and come in various shapes and sizes. These are super strong and lightweight cold forged aluminium alloys which can deteriorate with time and result in a variety of defects. Initial flaws are assumed to exist in these fracture-critical car rims. The fracture control requirement assumes a sharp crack as the initial flaw in the characterization of abnormal initial conditions. POD provides a framework and basis for development and assessment that support application and practice of risk assessment in car rims.

Practical use of fracture mechanics requires integration of nondestructive testing method as a crucial tool in design and service. Failure prediction techniques require the knowledge of the presence, size and location of potential defects within the car rims under stress. The POD treatment of complex fracture mechanics problem is presented based on theoretical simulation of the inspection process for the car rims. The results are good, satisfactory and can be regarded as reliable.

Introduction

The statistical model to be considered for this study is known as the probability of detection (POD). Ginzel [1] pointed out possible outcomes, namely, true and false positive as well as true and false negative, which are the foundations of the concept of probability of detection. Probability is the mathematical calculation of the likelihood that an event will occur. POD is used to establish the ability of inspections to detect cracks. In other words, POD describes how well a nondestructive testing procedure can detect the possible cracks.

Probability of detecting a crack will lie between 0 and 1 where 0 shows that there is no chance of detecting a crack while 1 shows that there is no chance of not detecting a crack. POD theory has been adopted widely [2, 3]. A POD can be used to assess a minimum flaw size that will be reliably detected by the nondestructive testing method. It can be established by the manufacturing of large numbers of realistic defected specimens. POD provides a framework and basis for development and assessment that support application and practice of risk assessment in car rims. To understand the use of POD, the accumulation of cracks detected is plotted against the crack size of all the cracks detected.

Aluminium alloys are used extensively in aerospace and automobile applications. They are broadly classified as either wrought or cast. They are usually alloyed with magnesium, copper, lithium, silicon, tin, manganese and zinc [4]. Wrought aluminium alloy is used in forging. Aluminium alloy is often strengthened through a variety of cold-working techniques that increase strength. Forging deforms the metal into a die cavity, producing relatively complex shapes such as automotive rims. The solidification process is used to manufacture specific components including aluminium alloy for automotive rims. Shrinkage and porosity can exist during solidification [5].

The car wheel is the result of the combination of the rim and center [6]. Different car rims are built for different purposes which can be for performance, looks or overall toughness. Cracked wheels may fail and come off the vehicle while it is moving which can result in serious injury or death. It was reported that the issue of cracked commercial vehicle wheels was raised [7]. It was also claimed that approximately take-off and second hand commercial vehicle wheels can enter a country aftermarket which many may be of substandard and even potentially hazardous [7].

Hair-like cracks can exist in exposed car rim surface. They can be very shallow or deep depending on the cause. Hair-like cracks are very fine cracks, usually running lengthwise along the shell and difficult to detect. Cracks can develop, especially where the spokes meet the outer car rim. This problem can be observed on polished wheels [6].

Rim failure may be due to improper strength, dimensions, chemical composition, abuse or overload [8]. Aluminium forged parts (car rims) must meet the highest safety standards against failure by abuse, shock, or vibratory stresses [9].

The fatigue striations (microscopic features) can be found in the cracks in the car rim. Sometimes a brief examination may easily reveal the fracture mode and cause of fracture. The entire car rim should be inspected for cracks, scrapes, gouges and upset areas [8].

1. Fracture Mechanics

Fracture Mechanics is the theory of the cracks contained by the materials and structures. It deals with the initiation and the propagation of the cracks. The car rims may deteriorate with time and result in a variety of defects. Initial flaws are assumed to exist in these fracture-critical car rims. The fracture control requirement assumes a sharp crack as the initial flaw in the characterization of abnormal initial conditions.

Practical use of fracture mechanics requires integration of nondestructive testing method as a crucial tool in design and service. Failure prediction techniques require the knowledge of the presence, size and location of potential cracks within car rims under stress. POD can be used as a useful tool in fracture mechanics.

1.1. 6061-O aluminium alloy fatigue

Fatigue is one of the primary reasons for the failure of car rims. It is a process that has a degree of randomness that often shows considerable scatter. Cracks may form as a result of fatigue. 6061 aluminium alloys are most widely used alloys in the 6000 series that feature magnesium and silicon as their primary alloying elements. Typical composition of 6061 aluminium alloy is given in Table 1 [10].

Table 1: Composition of 6061 aluminium alloy.
(Al = Aluminium, Mg = Magnesium, Si = Silicon, Fe = Iron, Cu = Copper, Zn = Zinc, Ti = Titanium, Mn = Manganese, Cr = Chromium).

Component	Al	Mg	Si	Fe	Cu	Zn	Ti	Mn	Cr	Others
Amount (w.t. %)	Balance	0.8-1.2	0.4-0.8	Max	0.7	0.15-0.40	Max 0.25	Max 0.15	0.4-0.35	0.05

Most 6061 aluminium alloys carry a supplemental temper (degree of hardness and elasticity) designation that shows whether the material is strain hardened or heat treated [5]. Thus, in the annealed condition, 6061-O aluminium alloy is extremely ductile and well suited for severe forming applications including car wheels and rims.

6061-O aluminium alloy has good mechanical properties, weldability, brazability, medium to high strength, toughness, surface finish, corrosion resistance to atmospheric conditions, corrosion resistance to sea water, workability and is widely available and can be anodized [10]. 6061-O aluminium alloy can be forged into all shapes including car wheels and rims. The mechanical properties for the 6061-O aluminium alloy are given in Table 2.

Table 2: Mechanical properties for the 6061-O aluminium alloy [11].

Material properties	Values
Hardness	30
Ultimate tensile strength	124 MPa
Tensile yield strength	55.2 Mpa
Modulus of elasticity	68.9 GPa
Fatigue strength	62.1 Mpa
Fracture toughness	42 MPa $\sqrt{m}$ [12]

Manufacturing automotive industries demand no flaw wheels and rims. However, during manufacturing and forming, cracking of car wheels and rims is a major problem at the production line.

1.2. Estimation of the critical crack length

In this paper, we assume that the car rims are invaded by cracks of length a subjected to uniaxial tension σ. Thus, the hoop stress is equal to 3σ [13]. We also assume the material element at the car rim when there is no crack. Due to stress concentration, the element is subjected to a tensile stress given as $\sigma' = 3\sigma$. Thus, the stress intensity factor can be given by

$$K_I = \beta\sigma'\sqrt{\pi a} \qquad (1)$$

where $\sigma' = 3\sigma$, σ is the applied stress, $\beta = 1.12$, is the geometry correction factor, I stands for Mode I loading and a is the crack length. In the linear elementary fracture mechanics, the stress intensity factor, K_I, is used as a measure for the crack driving force and the failure criterion is given as $K_I \geq K_{Ic}$ where c represents critical and K_{Ic} is the fracture toughness.

Fracture occurs when $K_I = K_{Ic}$ and $a = a_c$ where a_c represents the critical crack length. Critical crack length is the length at which a crack becomes unstable at a certain applied stress. This helps in determining material safety. Fracture toughness depends on the material, temperature, strain rate, environment and thickness. In some cases, experimental data can indicate that fracture toughness changes with temperature and strain rate. Thus, when the test temperature decreases, fracture toughness decreases. Solving for a_c from Eq. (1), it follows that

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\beta\sigma'}\right)^2 \qquad (2)$$

If we assume that the car rim is loaded so that $\sigma = \sigma_y$, where σ_y is the yield stress, then

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\beta(3\sigma_y)}\right)^2 \quad (3)$$

Fracture Mechanics allows analysis of critical crack length for unstable crack growth loading to fracture. Thus, car rims must be inspected periodically to ensure that actual crack lengths are smaller than critical crack lengths, otherwise, the car rims have the potential to fail.

2. Brief statistical analysis

Nondestructive testing of car rims can effectively be done by applying liquid penetrant testing, magnetic particle testing, visual testing, radiographic testing, eddy current testing, and ultrasonic testing methods. Therefore, the probability of finding cracks in car rims is crucial and necessary.

It is often useful to create a model using simulation. Usually, this takes the form of generating a series of random observations, often based on a specific statistical distribution, then study the resulting observations. Since we require random variables (X) to represent crack lengths for our hypothetical experiment, the data set may be acquired by using the command randi given by [14, 15]

$$X = randi\ ([a_i\ \ a_c], 1, m) \quad (4)$$

where X is the random variable representing a set of crack lengths, a_i is the minimum crack length, a_c is the critical crack length and m represents the number of crack rims. The command randi generates a 1-by-m row vector of uniformly distributed random integers from the sample interval [a_i, a_c]. The command randi accepts many parameter sets but we chose the one shown in Eq. [5] and modified it to suit our purpose of study. This formula is appropriate because it can specify the range and random integers from the uniform distribution between a_i and a_c.

The probability of the occurrence of a particular event E is given by [16]

$$P(E) = \frac{Number\ of\ ways\ E\ can\ occur}{Total\ number\ of\ possible\ outcomes} \quad (5)$$

A random variable X has the lognormal distribution with parameters $\mu \in R$ and $\alpha > 0$ if $\ln X$ has the normal distribution with mean μ and standard deviation α. A lognormal distribution (lognormal probability density (PDF)) is a probability distribution with a normally distributed logarithm. The lognormal distribution can be used to model the life of a component (car rim) whose failure mode is of a fatigue-stress nature [17].

A PDF could be fitted to crack length results and can be used to model the cycle to failure for a car rim. This PDF is given by [18]

$$PDF(\ln X, \mu, \alpha) = \frac{1}{X\alpha\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln X - \mu}{\alpha}\right)^2\right], \qquad X > 0, \alpha > 0, -\infty < \mu < +\infty \quad (6)$$

where μ and α are the location and shape parameters of the lognormal distribution, respectively. The parameters μ and σ of the normal PDF are written in the form [19]

$$\mu = \log\left(\frac{k^2}{\sqrt{v+k^2}}\right) \tag{7}$$

and

$$\sigma = \sqrt{\log\left(\frac{v}{k^2}+1\right)} \tag{8}$$

where k is the mean and v is the variance of the lognormal random variable. Since we do not know the population mean k and variance v, we can estimate μ and σ as follows [19]

$$\mu = \text{mean } (\log X) \tag{9}$$

and

$$\sigma = \text{std } (\log X) \tag{10}$$

where $\log X > 0$ and std stands for standard deviation.

The POD can be used as the basis for characterization and validation of the quantified detection for a nondestructive evaluation procedure. The detection of crack lengths is modelled using a POD curve, which describes the probability of detecting a crack with a specific length. Therefore, nondestructive inspection quality for the individual car rims can be modelled by a POD curve given by [20]

$$\mathrm{P(D|\,x)} = A\left[1-\exp\left(-\frac{X-a_i}{a_1-a_i}\right)\right] \tag{11}$$

where x is a crack (flaw), a_i is the initial crack length, a_1 is any crack length immediately after the initial crack length and A is the amplitude of the POD curve.

To effect the POD, we let $P(D)$ represent the probability of detection of a crack, $P(D|\,a_i < X \le b_i)$ the probability of existence of any crack in the size range $(a_i < X \le b_i)$ given by Eq. (11), $P(a_i < X \le b_i)$ the probability that a variable X is between a_i and b_i, $P(X)$ the probability of existence of any crack and a and b the lower and upper bounds of each interval *i* respectively. Thus, the probability of detecting cracks in car rims may be given by [21]

$$P(D) = P(D|\,a_i < X \le b_i)P(a_i < X \le b_i)P(X) \tag{12}$$

If we sum over the interval containing the crack distribution, then Eq. (12) can be expressed in the form [21]

$$P(D) = \left[\sum_x P(D|x) \right] P(X) \tag{13}$$

where

$$\sum_x P(D \mid x) = \sum_x \left[P(D \mid a_i < X \leq b_i) P(a_i < X \leq b_i) \right] \tag{14}$$

is the probability of locating a crack within the crack distribution and $P(X)$ is the probability of any car rim having a crack.

3. Hypothetical experiment

The effectiveness of various inspection methods is characterized by probability of detection curves [2, 22]. In this case, we consider a number of m forged polished car rims that have been brought for inspection by the fictitious XG wheel company. The car rims to be inspected have different crack lengths.

The hypothetical experimental data set that will mimic the dimensions of the fatigue cracks (hair-like cracks) is determined together with their number of occurrences in the car rims. The dimensions of fatigue cracks can be considered to fill an infinite sample space. To realize this, numerical categories are defined and the fatigue crack data set is introduced within these limits. The survey of a group of forged polished car rims is done by using Eq. (5) to obtain random variables as shown in Table 3 for existing fatigue cracks. It can be assumed that a crack length distribution, as shown in Table 3, may be expected for a particular car rim design.

The hypothetical experiment consists of the sample of $m =$ 120 car rims to be inspected using a nondestructive testing method which has the required crack detection capabilities. Seven (7) crack lengths are assumed to be contained in this sample and we need to find the probability of detecting a crack length in the inspection of a single car rim selected randomly from the sample. We need also to locate crack lengths and find out how many were missed in the inspection when all the car rims have been inspected without replacement.

4. Results and discussion

The nondestructive testing method which results in the larger crack lengths being detected and smaller crack lengths being missed in the car rims, can be depicted in Fig. 2. Table 3 gives the numerical categories that allow the fatigue crack data to be allocated within the intervals. We assumed that the sample of car rims could result in the data as shown in Table 3 for the existing fatigue cracks. The cracks contained within each category gives the probability of crack indicated by $P(a_i < X \leq b_i)$, which means the number of cracks in the category is divided by the total number of predetermined cracks. $P(a_i < X \leq b_i)$ is made possible by determining the number of occurrences of the cracks within each category.

Table. 3: Distribution of crack lengths.

Dimensions	Number of Occurrences	P($a_i < X \leq b_i$)
0<x<2	6	0.00293
$2 \leq x < 4$	33	0.01632
$4 \leq x < 6$	53	0.02621
$6 \leq x < 8$	122	0.06034

$8 \leq x < 10$	111	0.05490
$10 \leq x < 12$	145	0.07171
$12 \leq x < 14$	260	0.12859
$14 \leq x < 16$	233	0.11523
$x \leq 16$	1059	0.52374
Total	2022	1.00001

In Table 4, the results obtained by using the equations provided in this paper are outlined. The results emanate from the simulation of the experiment conducted using Eq. (5). The initial crack length was assummed to be detected by the nondestructive testing methods and the critical crack length was calculated using Eq. (3). The literature values of the fracture toughness K_{Ic} and the tensile strength σ of the 6061-O aluminium alloy have been included in Table 4.

Table 4: The results as obtained from the calculations.

Mean	1.98
Standard deviation	0.72
Initial crack length	$1mm$
Critical crack length	16 mm
Number of cracks contained in thesample of 120 rims	7 cracks
Probabilty of locating a crack	0.9612 $(\approx 96.1\%)$
Probability of finding a crack	0.0561 $(\approx 5.6\%)$
Number of cracks found	5.6072 (≈ 6)
Number of cracks missed	1.3928 (≈ 1)

Eq. (6) was employed to calculate probability density function as shown in Fig. 1. The random values calculated from the command randi were considered as crack lengths less than the critical crack length.

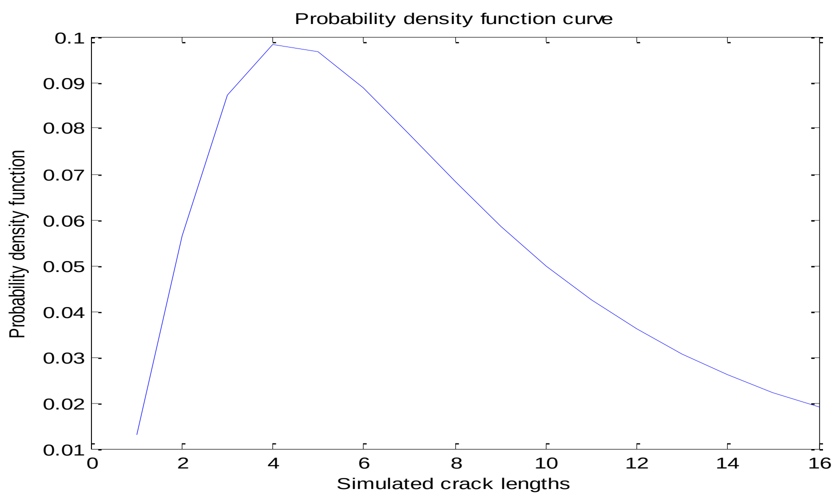

Fig. 1: Crack distribution versus simulated crack lengths in mm obtained by using Eqs. (5) and (6) where $\mu = 1.9813$ and $\alpha = 0.7234$.

These values were used to obtain the mean and standard deviation of the lognormal distribution. Fig. 1 shows the probability density function against the simulated crack lengths. The curve is asymmetric (skewed to the right). It starts at zero, increases to its mode and decreases to its median down to its mean. It can be observed that the mode is less than the median while the median is less than the mean. This implies that the values are clustered around low values.

Eq. (11) was used as a reference to what the probability of detection would look like when the nondestructive testing procedure is efficient. The curve of Fig. 2 shows the initial crack length that can be detected by the nondestructive testing method with a zero percent probability of detection for initial crack length while for the larger crack lengths, the percentage of probability of detection is much higher.

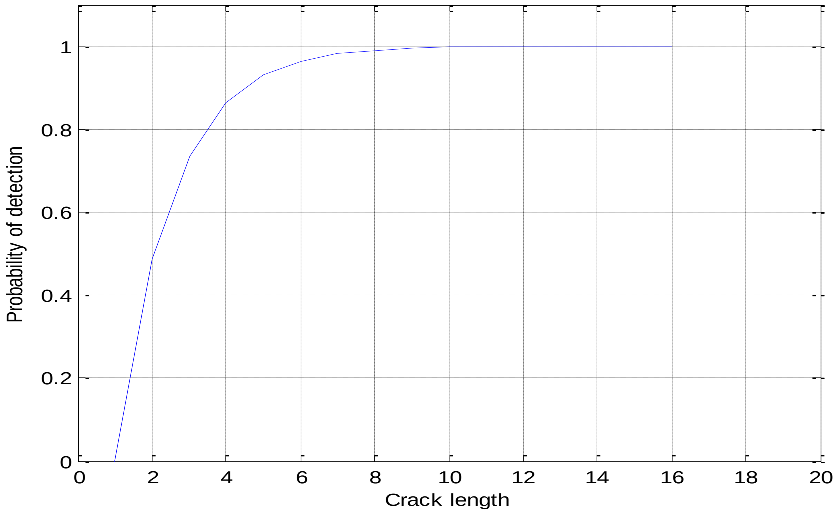

Fig. 2: Probability of detection against crack lengths in mm using Eq. (11) for the inspection of car rims where $a_i = 1mm$ and $a_1 = 2.5mm$.

5. Conclusion

Failures of car wheels and rims have been known to be detected by the use of nondestructive testing methods. The simulated cracks in the car rims mimic those that occur as a result of fatigue. The lengths of these cracks were controlled by first determining the possible critical crack length that can be found for 6061-O aluminium alloys using Fracture Mechanics.

The POD curve was estimated based on theoretical simulation of the inspection process for the car rims using the command randi. The inspection quality for the car rims was also modeled by POD curve. The hypothetical experimental results presented in this paper look good, satisfactory and can be regarded as reliable.

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